This Algebra I Study Guide contains clear, straight-forward problems that represent the topics covered in a complete Algebra I course. After completing the study guide without a calculator, correct it with the Solution Guide. If there is a topic that was difficult for you, you should use your textbook to practice similar problems.

Algebra I Study Topics

1. Rational Expressions

2. Integers

3. Order of Operations

4. Prime Numbers

5. Graphing Numbers and Inequalities of Real Numbers

6. Graphing and Labeling Ordered Pairs on a Coordinate (Cartesian) Plane

7. Relations

8. Exponents

9. Proportions

10. Absolute Values

11. Radical Expressions

12. Scientific Notation

13. Translating Words into Symbols

14. Algebraic Expressions

15. Solving Equations

16. Literal Equations

17. Pythagorean Theorem

18. Monomials

19. Polynomials

20. Factoring

21. Algebraic Fractions

22. Quadratic Equations

23. Solving and Graphing Inequalities

24. Graphing Linear Equations

25. Slope

26. Writing Equations of Lines

27. Functions

28. Systems of Equations

29. Graphing Systems of Inequalities

30. Word Problems
    a) Ratio
    b) Consecutive Numbers
    c) Direct Variation
    d) Indirect Variation
    e) Age
    f) Percent Solution
    g) Motion
    h) Rate of Work
    i) Linear
    j) Area
    k) Discount

Name ___Key_____ Grade _____ Date _____

School _____ Teacher _____

1. Rational Expressions - simplify

a. $8 \frac{1}{6} + 5 \frac{3}{4}$

$13 + \frac{4}{24} + \frac{18}{24} = 13 \frac{22}{24} = \boxed{13 \frac{11}{12}}$

b. $7 \frac{1}{2} - 2 \frac{7}{10}$

$7 \frac{5}{10} - 2 \frac{7}{10}$

$6 \frac{15}{10} - 2 \frac{7}{10} = 4 \frac{8}{10} = \boxed{4 \frac{4}{5}}$

c. $4 \frac{2}{3} \cdot 7 \frac{1}{2}$

$\frac{14}{3} \cdot \frac{15}{2} = \boxed{35}$

d. $4 \frac{2}{5} \div 3 \frac{2}{10}$

$\frac{22}{5} \div \frac{32}{10} = \frac{22}{5} \cdot \frac{10}{32} = \boxed{\frac{11}{8}}$

2. Integers - simplify

a. $-2 + 4 + (-3) + 1$

$-5 + 5 = \boxed{0}$

b. $5 - (-3) - 2$

$5 + 3 - 2 = \boxed{6}$

c. $-2 \left(\frac{1}{2}\right) (-3)$

$(6)\left(\frac{1}{2}\right) = \boxed{3}$

d. $(-10 \div 5) \div 2$

$-2 \div 2 = \boxed{-1}$

3. Order of Operations - simplify

a. $3 + 6 \div 2 \cdot 3$

$$3 + 3 \cdot 3$$
$$3 + 9 = \boxed{12}$$

b. $(5\frac{1}{5} - 2\frac{1}{5}) - 6 \cdot \frac{1}{2}$

$$3 - 3 = \boxed{0}$$

c. $\dfrac{2 + 5 \cdot 2}{7 - 20 \div 4}$

$$\frac{12}{2} = \boxed{6}$$

4. Prime Numbers

a. Define "prime number"

A positive natural number greater than one that is only divisible by itself and one.

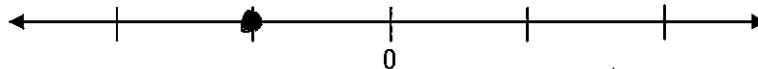b. List the first five prime numbers.

2, 3, 5, 7, 11

c. Write the prime factors of 200.

$$2 \cdot 10 \cdot 10 = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \quad or \quad 2^3 \cdot 5^2$$
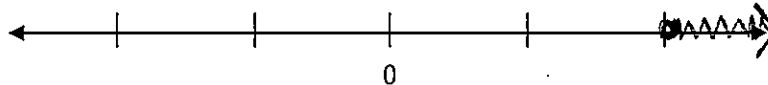$$2 \ and \ 5$$

5. Graphing Numbers and Inequalities of Real Numbers – graph the following
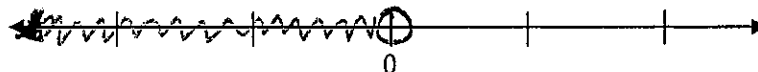
a. $x = -1$
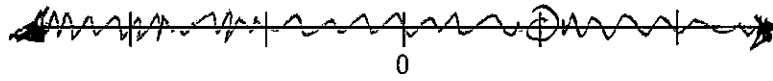

b. $x \geq 2$

c. $x < 0$

5. Graphing Numbers and Inequalities of Real Numbers – graph the following (cont'd)
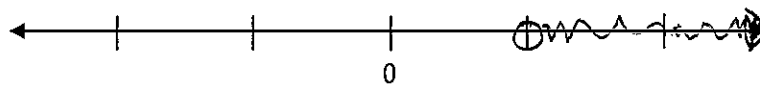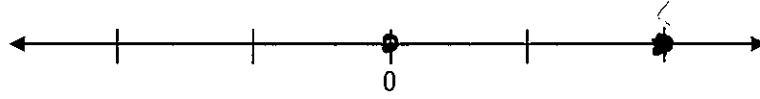
d. $x \neq 1$

e. $|x - 1| = 1$

$X - 1 = 1$
$X = 2$

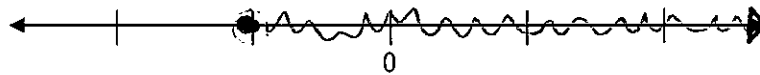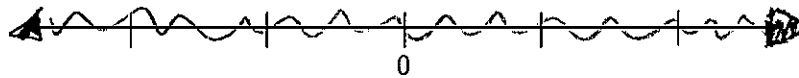$X - 1 = -1$
$X = 0$

f. $x + 3 > 4$
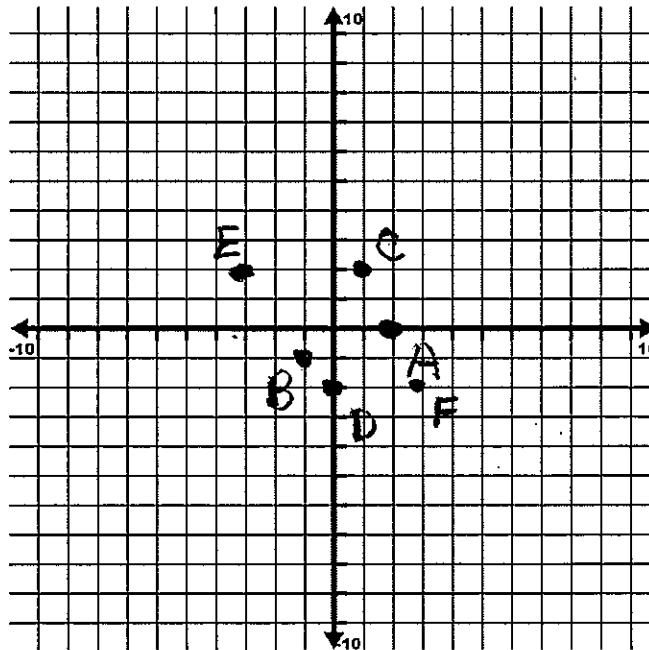
$X > 1$

g. $-2x \leq 2$

$X \geq -1$

h. $|x + 1| \geq -1$

all numbers

6. Graphing and Labeling Ordered Pairs on a Coordinate (Cartesian) Plane – graph the following

A (2, 0), B (-1, -1), C (1, 2), D (0, -2), E (-3, 2), F (3, -2)

7. Relations - state the domain and range of each relation. Is the relation a function?

a. $\{(3,4), (2,3), (3,6), (4,1)\}$

b. $\{(1,0), (2,0), (3,0), (4,0)\}$

domain 2, 3, 4
range 1, 3, 4, 6
it's a relation, not a function

domain 1, 2, 3, 4
range 0
This is a function

8. Exponents - simplify

a. $2(2)^3$

$2 \cdot 8 = \boxed{16}$

b. $(-3)^2 - (-1)^3$

$9 - (-1) = \boxed{10}$

c. -5^{-2}

$= \dfrac{-1}{5^2} = \boxed{-\dfrac{1}{25}}$

d. $3x^2 \cdot x^{-1}$

$\boxed{3x}$

9. Proportions – solve for the variable

a. $8:12 = 4:3x$

$\dfrac{8}{12} = \dfrac{4}{3x}$ $24x = 48$
$\boxed{x = 2}$

b. $\dfrac{3n}{20} = \dfrac{3}{5}$

$15n = 60$
$\boxed{n = 4}$

10. Absolute Values - simplify

a. $|5 + (-2)| \div 3$

$|3| \div 3 = 3 \div 3 = \boxed{1}$

b. $-(3|2|) + -(|2|)$

$-6 + (-2) = \boxed{-8}$

c. $-(4|-3| + |-6|)$

$-(12 + 6) = \boxed{-18}$

11. Radical Expressions – Simplify

a. $\sqrt{9} + \sqrt{16}$

$3 + 4 = \boxed{7}$

b. $\sqrt{200}$

$100 \cdot 2$

$\boxed{10\sqrt{2}}$

c. $2\sqrt{3} + \sqrt{3}$

$\boxed{3\sqrt{3}}$

12. Scientific Notation

Write in scientific notation:

a. 32,000,000

$\boxed{3.2 \times 10^7}$

b. 0.000012

$\boxed{1.2 \times 10^{-5}}$

Write in standard form:

c. 4.1×10^{-3}

$.004.1 \quad \boxed{.0041}$

d. 6.3×10^4

$63.000 \quad \boxed{63,000}$

13. Translating Words into Symbols – translate the following statements into an algebraic expression

a. Eight less than one third of x.

$\boxed{\frac{1}{3}x - 8} \quad or \quad \frac{x}{3} - 8$

b. Twice a number, increased by six.

$\boxed{2x + 6}$

c. Six, decreased by six times a number.

$\boxed{6 - 6x}$

14. Algebraic Expressions - simplify

    a. $6x - 3y + 14x - 7y$ $\boxed{20x + 4y}$

    b. $4(n + 7) + 5(n - 3) - 2n$

$$4n + 28 + 5n - 15 - 2n = \boxed{7n + 13}$$

    c. $-3(7c + d) - 2(10d - c)$

$$-21c - 3d - 20d + 2c$$
$$\boxed{-19c - 23d}$$

Evaluate the expression if $w = \frac{1}{2}$, $x = 3$, and $y = -4$

    a. $wy + 2x$

$$\left(\tfrac{1}{2}\right)(-4) + 2(3)$$
$$-2 + 6 = \boxed{4}$$

    b. $2w(3x - 2y) + 4w$

$$1(9 + 8) + 2$$
$$17 + 2 = \boxed{19}$$

15. Solving Equations – solve for the variable

    a. $5x + 3 = 18$

$$5x = 15$$
$$\boxed{x = 3}$$

    b. $\frac{1}{2}x - 1 = 3$

$$\tfrac{1}{2}x = 4$$
$$\boxed{x = 8}$$

    c. $8x - x + 1 = 8$

$$7x = 7$$
$$\boxed{x = 1}$$

    d. $2(x + 8) - 9 = 5$

$$2x + 16 - 9 = 5$$
$$2x + 7 = 5$$
$$2x = -2$$
$$\boxed{x = -1}$$

15. Solving Equations – solve for the variable (cont'd)

e. $3(x + 5) - 6 = 3(x + 3)$

$3x + 15 - 6 = 3x + 9$

$9 = 9$

(ℝ) (all real numbers)

f. $\frac{6 - 4y}{2} = y$

$6 - 4y = 2y$

$6 = 6y$

$y = 1$

g. $\left[\frac{3x}{10} + \frac{x}{5} = \frac{3}{2}\right] 10$

$3x + 2x = 15$

$5x = 15$

$x = 3$

h. $\left[\frac{4x + 1}{3} - \frac{2x + 1}{5} = \frac{3}{5}\right] 15$

$5(4x + 1) - 3(2x + 1) = 9$

$20x + 5 - 6x - 3 = 9$

$14x + 2 = 9$

$14x = 7$

$x = \frac{1}{2}$

i. $\frac{3}{x - 2} = \frac{6}{x + 3}$

16. Literal Equations – solve for the underlined variable

a. $C = \underline{\pi} r^2$

$\pi = \frac{C}{r^2}$

b. $y = \underline{m}x + b$

$y - b = \underline{m}x$

$m = \frac{y - b}{x}$

c. $P = 2\underline{l} + 2w$

$P - 2w = 2\underline{l}$

$l = \frac{P - 2w}{2}$
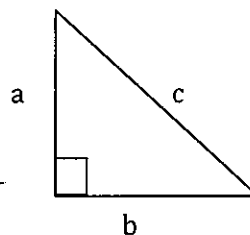
17. Pythagorean Theorem – state and solve the equation for the length of the unknown side

a. State the Pythagorean Theorem

$$a^2 + b^2 = c^2$$

The sum of the legs squared of a right triangle equals the hypotenuse squared.

b. $a = 6$, $b = 8$, $c = \underline{\hphantom{10}}$

$$36 + 64 = 100 = c^2 \qquad \boxed{c = 10}$$

c. $a = 3$, $b = \underline{\hphantom{4}}$, $c = 5$

$$9 + b^2 = 25$$
$$b^2 = 16$$
$$\boxed{b = 4}$$

18. Monomials – simplify

a. $(3a^2)(4a^3)$

$\boxed{12a^5}$

b. $(-3a^2b^5)^2$

$\boxed{9a^4b^{10}}$

c. $(2a)^2(3y)$

$$4a^2(3y) = \boxed{12a^2y}$$

d. $(5y)^2 + (3y)(7y)$

$$25y^2 + 21y^2 = \boxed{46y^2}$$

e. $\dfrac{a^3b^4}{a^2b^2}$ $\boxed{ab^2}$

f. $\dfrac{(a^7b^2)^2}{(a^{-2}b)^{-2}}$

$$\dfrac{a^{14}b^4}{a^4b^{-2}} = \boxed{a^{10}b^6}$$

g. $\left(\dfrac{3m^2n^2}{6m^{-1}k}\right)^0$ $= \boxed{1}$

19. Polynomials – simplify

$-x^2 + 5x - 1$

a. $(2x^2 - 3x + 2) - (x^2 - 5x + 1) + (x^3 + x + 3)$

$x^3 + x^2 + (-3x + 5x + x) + (2 - 1 + 3)$

$x^3 + x^2 + 3x + 4$

b. $3xy^2(2x^2 + 3x + 4)$

$6x^3y^2 + 9x^2y^2 + 12xy^2$

c. $(2x^2 + 1)(x - 3)$

$2x^3 - 6x^2 + x - 3$

d. $(3x - 1)(3x - 1)$

$9x^2 - 6x + 1$

e. $\dfrac{6x^3y + 3x^2y + 12xy}{3xy}$

$2x^2 + x + 4$

20. Factor Completely

a. $18xy^2 - 24x^2y$

$6xy(3y - 4x)$

b. $4x^2 - 9y^2$

$(2x + 3y)(2x - 3y)$

c. $b^2 + 10b + 25$

$(b + 5)(b + 5)$

or

$(b + 5)^2$

d. $x^2 - 9x + 14$

$(x - 7)(x - 2)$

20. Factor Completely (cont'd)

e. $y^2 - 7y - 30$

$(y-10)(y+3)$

f. $2ax + 6xc + ba + 3bc$

$2x(a+3c) + b(a+3c)$

$(a+3c)(2x+b)$

g. $3x^2 + 7x - 6$

$(3x-2)(x+3)$

21. Algebraic Fractions – simplify and find restrictions

a. $\dfrac{x^2 - 3x}{x^2 - 2x - 3}$

$\dfrac{x(x-3)}{(x-3)(x+1)}$

$\dfrac{x}{x+1} \quad x \neq 3, -1$

b. $\dfrac{3x^2 + 3x}{x^2 - 5x - 6}$

$\dfrac{3x(x+1)}{(x-6)(x+1)}$

$\dfrac{3x}{x-6} \quad x \neq 6, -1$

22. Quadratic Equations

Solve using the zero principal

a. $y^2 - 16 = 0$

$(y+4)(y-4) = 0$

$y = -4 \quad y = 4$

b. $2x^3 + 8x^2 = -8x$

$2x^3 + 8x^2 + 8x = 0$

$2x(x^2 + 4x + 4) = 0$

$2x(x+2)(x+2) = 0$

$x = 0 \quad x = -2$

22. Quadratic Equations (cont'd)

c. $\left[1 = \dfrac{3}{x+2} + \dfrac{1}{x-2}\right](x+2)(x-2)$

$x^2 - 4 = 3(x-2) + (x+2)$

$x^2 - 4 = 3x - 6 + x + 2$

$x^2 - 4x = 0 \qquad x(x-4) = 0$

$\boxed{x = 0 \qquad x = 4}$

Quadratic Formula
    d.   State the quadratic formula

$$\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve using the quadratic formula

$a = 1 \quad b = 7 \quad c = 6$

e. $x^2 + 7x + 6 = 0$

$$\dfrac{-7 \pm \sqrt{49 - 24}}{2}$$

$$\dfrac{-7 \pm \sqrt{25}}{2} = \begin{array}{l}\nearrow \ \frac{-2}{2} \\ \searrow \ \frac{-12}{2}\end{array}$$

$\boxed{x = -1 \quad x = -6}$

$a = 1 \quad b = 1 \quad c = -12$

f. $x^2 + x = 12$

$x^2 + x - 12 = 0$

$$\dfrac{-1 \pm \sqrt{1 + 48}}{2} = \dfrac{-1 \pm \sqrt{49}}{2} \begin{array}{l}\nearrow \ +\frac{6}{2} \\ \searrow \ -\frac{8}{2}\end{array}$$

$\boxed{x = 3 \quad x = -4}$

Solve by graphing - find roots (x-intercepts)

g. $x^2 + x - 6 = 0$

$\boxed{\text{x-intercepts } -3 \text{ and } 2}$

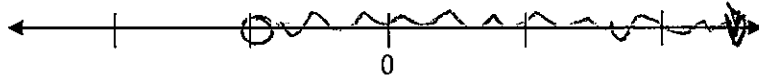| x		y
-3	$9 - 3 - 6$	0
-2	$4 - 2 - 6$	-4
-1	$1 - 1 - 6$	-6
0	$0 - 0 - 6$	-6
1	$1 + 1 - 6$	-4
2	$4 + 2 - 6$	0
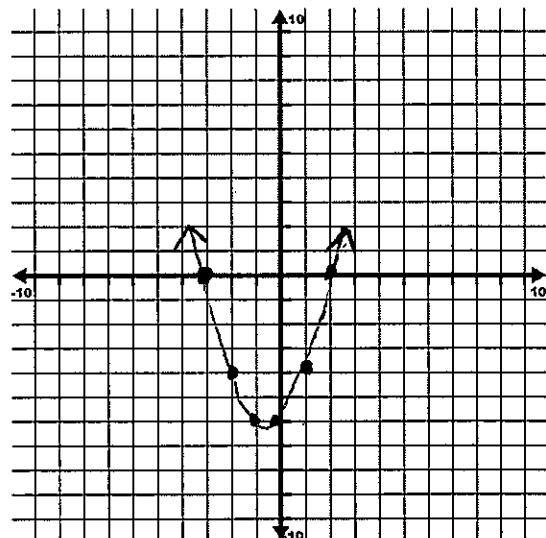
$\dfrac{-1}{2}$

23. Solving and Graphing Inequalities – solve and graph the solution

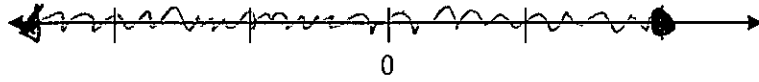a. $5y + 4 > 2y + 1$

$$3y > -3$$
$$y > -1$$

b. $-3(2x - 7) \geq 4x - (x - 3)$

$$-6x + 21 \geq 4x - x + 3$$
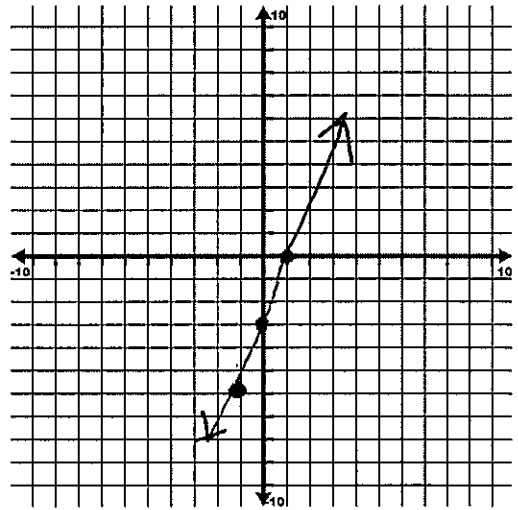$$-9x \geq -18 \qquad x \leq 2$$

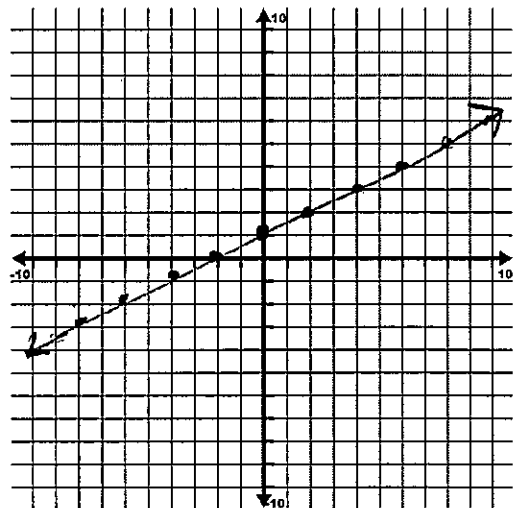## 24. Graphing Linear Equations

a. using table: $3x - y = 3$

$-y = -3x + 3$

x	$3x - 3$	y
-1	$-3 - 3$	-6
0	$0 - 3$	-3
1	$3 - 3$	0

b. using slope-intercept form:
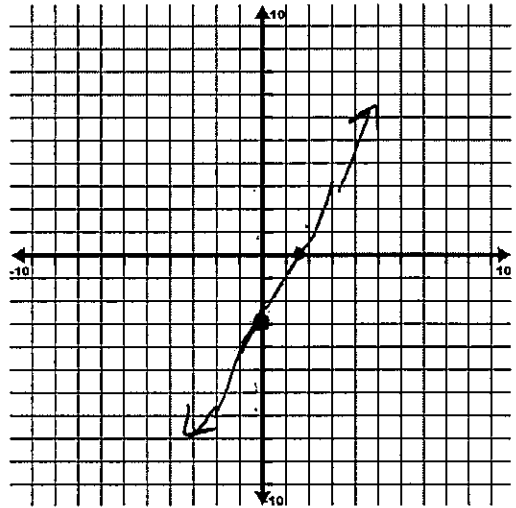$y = \frac{1}{2}x + 1$

24. Graphing Linear Equations (continued)

c. using intercepts: $4x - 2y = 6$

$$4x = 6$$
$$x = 1\tfrac{1}{2}$$

$$-2y = 6$$
$$y = -3$$

25. Slope – find the slope

a. $y = 3x - 1$

$$m = 3$$

b. $2y = 5$

$$m = 0$$

c. $x = 6$

no slope or undefined

25. Slope – find the slope (cont'd)
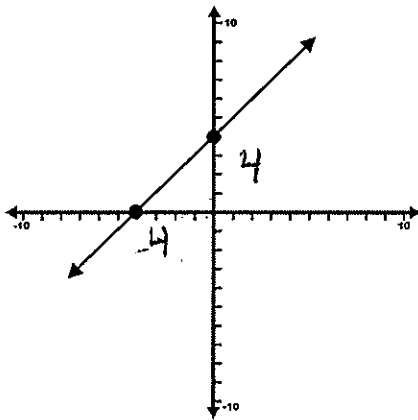
    d. $10x + 2y = 4$

$$2y = -\frac{10}{2}x + 4$$

$$\boxed{m = -5}$$

    e. $(2, -1)\ (-1, 0)$

$$m = \frac{0+1}{-1-2} = \frac{1}{-3} \qquad \boxed{m = -\frac{1}{3}}$$

    f.

$$\boxed{m = 1}$$

    g.   line parallel to $y = 7x - 1$

$$\boxed{m = 7}$$

    h.   line perpendicular to $y = \frac{3}{2}x + 6$

$$\boxed{m = -\frac{2}{3}}$$

26. Equations of a Line – write an equation for the line described using <u>both</u> point-slope and slope-intercept form

a. $m = 3, b = 1$ $(0, 1)$

$$y = 3x + 1$$
$$y - 1 = 3(x - 0)$$

b. $m = 2$, passing through point $(4, -2)$

$$y + 2 = 2(x - 4)$$
$$y = 2x - 10$$

c. passing through $(4, 1)$ and $(5, 2)$

$$m = \frac{2 - 1}{5 - 4} = \frac{1}{1} = 1$$

$$y - 1 = 1(x - 4) \quad \text{or} \quad y - 2 = 1(x - 5)$$
$$y = x - 3$$

d. parallel to $x + y = 2$, passing through $(1, 2)$

$$y = -x + 2 \qquad m = -1$$

$$y - 2 = -(x - 1)$$
$$y = -x + 3$$

e. perpendicular to $x - 4y = 16$, passing through $(-1, 1)$

$$-4y = -x + 16 \qquad y = \frac{1}{4}x - 4$$

$$y - 1 = \frac{1}{4}(x + 1)$$
$$y = \frac{1}{4}x + \frac{5}{4}$$

27. Functions

Find the range of the given function:

a. $H: b \to b^2 + 3, D = \{-1, 0, 2\}$

$-1: \quad 1 + 3 = 4$

$0: \quad 0 + 3 = 3$

$2: \quad 4 + 3 = 7$

Range : $\{3, 4, 7\}$

Find the values for each given function with the set of Real numbers as the domain:

$g: x \to 2x - 1 \quad h: y \to y^2 + 1$

b. $g(0) = 2(0) - 1 = \boxed{-1}$

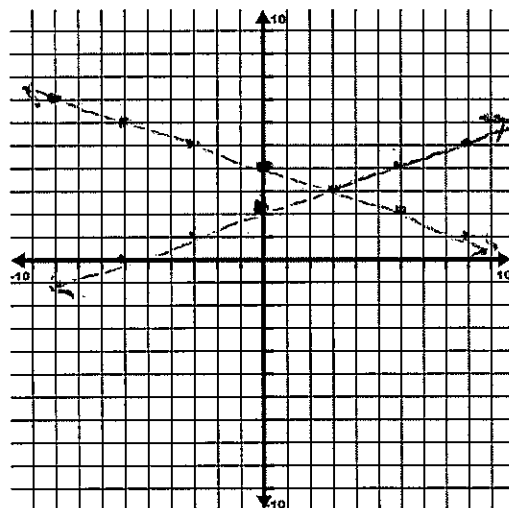c. $g(-1) = 2(-1) - 1 = \boxed{-3}$

d. $h(2) = 4 + 1 = \boxed{5}$

e. $h(-3) = 9 + 1 = \boxed{10}$

28. Systems of Equations – solve, then determine whether the system has one solution, no solution, or an infinite number of solutions

a. Graphically $y = -\frac{1}{3}x + 4$ and $y = \frac{1}{3}x + 2$

$(3, 3)$

one solution

28. Systems of Equations – solve, then determine whether the system has one solution, no solution, or an infinite number of solutions (continued)

b. Substitution $x = 3 - 2y$ and $2x + 4y = 6$

$$2(3 - 2y) + 4y = 6$$
$$6 - 4y + 4y = 6$$
$$6 = 6$$

infinite number of solutions

c. Elimination using addition $2x - 3y = -4$ and $x = 7 - 3y$

$$2x - 3y = -4$$
$$x + 3y = 7$$
$$\overline{}$$
$$3x = 3$$
$$x = 1$$

$$2 - 3y = -4$$
$$-3y = -6$$
$$y = 2$$

$(1, 2)$ one solution

d. Elimination using multiplication $3x + 3y = 6$ and $(2x - y = 1) \, 3$

$$3x + 3y = 6$$
$$6x - 3y = 3$$
$$\overline{}$$
$$9x = 9$$
$$x = 1$$

$$3 + 3y = 6$$
$$3y = 3$$
$$y = 1$$

$(1, 1)$ one solution

29. Graphing Systems of Inequalities – solve by graphing

$y \le 3x + 3$ and $y > -\frac{2}{3}x$
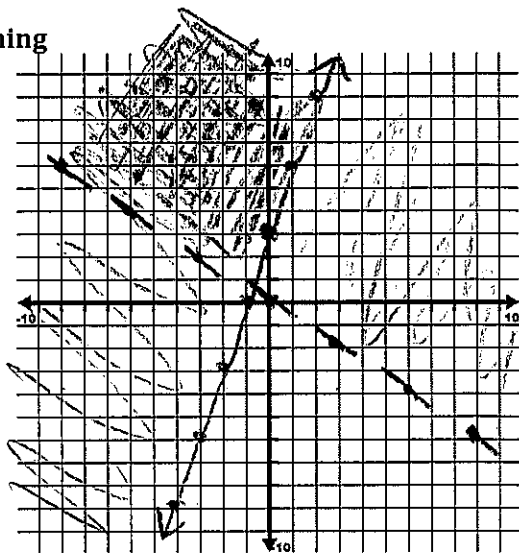
30. Word Problems

a. <u>Ratio</u> - How many of the 28 members of the Math Team are boys if the ratio of girls to boys is 2 to 5?

$$\frac{girls}{boys} \quad \frac{2}{5} \qquad \frac{boys}{all} \quad \frac{5}{7} = \frac{b}{28} \qquad \boxed{b = 20}$$

b. <u>Consecutive Numbers</u> - Find two consecutive odd integers whose sum is 64.

$$x \qquad \#1$$
$$x+2 \qquad \#2$$

$$2x + 2 = 64$$
$$2x = 62$$
$$\boxed{\begin{array}{c} x = 31 \\ x+2 = 33 \end{array}}$$

c. <u>Direct Variation</u> - If y varies directly as x, and $y = 6$ when $x = 8$, find y when $x = 12$.

$$y = kx \qquad \frac{y}{x} = k$$

$$\frac{6}{8} = \frac{y}{12} \qquad 8y = 72$$
$$\boxed{y = 9}$$

d. <u>Indirect Variation</u> - If y varies inversely as x, and $y = 6$ when $x = 12$, find x when $y = 9$.

$$y = \frac{k}{x} \qquad xy = k$$
$$(12)(6) = 9x$$
$$72 = 9x$$
$$\boxed{x = 8}$$

30. Word Problems (continued)

e. <u>Age</u> - Mike is 3 years older than Will. The sum of their ages in 4 years will be 59 years. How old is Mike now?

Mike   now   in 4yr

	now	in 4yr
Mike	$x+3$	$x+7$
Will	x	$x+4$

$$2x + 11 = 59$$
$$2x = 48$$
$$x = 24$$

$$\boxed{\text{Mike now is } x+3 \text{ or } 27}$$

f. <u>Solution</u> - How much pure orange juice would have to be added to 5 liters of a 10% orange juice solution to obtain a mixture containing 40% orange juice?

Juice	% orange	orange
x	100	x
5	.10	.5
$x+5$	.40	$.4x+2$

$$x + .5 = .4x + 2$$
$$.6x = 1.5$$
$$\boxed{x = 2.5 \text{ liters}}$$

g. <u>Motion</u> - Clark and Kent leave home traveling on their bicycles in opposite directions. Clark travels 10km/h and Kent travels 12km/h. In how many hours will they be 110 km apart?

Kent   Clark

←————•————→
12t   10t

$$12t + 10t = 110$$
$$22t = 110$$
$$\boxed{t = 5 \text{ hours}}$$

h. <u>Rate of Work</u> - Peter can do a job in 10 hours, while Parker can do the same job in 15 hours. How long will it take them to complete the job if they work together?

	R	· T	= W
Peter	$\frac{1}{10}$	x	$\frac{x}{10}$
Parker	$\frac{1}{15}$	x	$\frac{x}{15}$

complete job = 1

$$\left[\frac{x}{10} + \frac{x}{15} = 1\right]30$$
$$3x + 2x = 30$$
$$5x = 30$$
$$\boxed{x = 6 \text{ hours}}$$

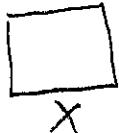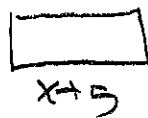### 30. Word Problems (continued)

i. <u>Linear Rate of Growth</u> - A plant is 2 inches tall and it grows at a rate of ½ in per week. Write an equation that models the height, h, in inches, of the plant with respect to time, t, in weeks.

$$h = \tfrac{1}{2}t + 2$$

j. <u>Area</u> – There are two rooms of equal area. One room is square and the other is a rectangle 4 ft narrower and 5 ft longer than the square one. Find the area of each room.

$$x^2 = (x+5)(x-4)$$
$$x^2 = x^2 + x - 20$$
$$20 = x$$

Area of each room is 400

k. <u>Discount</u>
1. Two bikes are on sale at the bike shop. A red bike originally cost $280 and has a 15% discount and a blue bike originally cost $300 and has a 20% discount. Which bike will cost less? How much less than the other bike does it cost?

Red $(280)(.85) = \$238$

Blue $(300)(.80) = \$240$

The Red bike costs less by $2

2. A $120 pair of running sneakers is on sale for $96. What is the percent discount?

$$\frac{discount}{100} = \frac{24}{120}$$

$$120d = 2400$$

$$d = \frac{2400}{120}$$

$$\begin{array}{r} 120 \\ -96 \\ \hline 24 \end{array}$$

percent discount is 20%